Chapter 4: Discrete Probability Distributions

Q1. A survey of cars on a certain stretch of highway during morning commute hours showed that 70% had only one occupant, 15% had 2, 10% had 3, 3% had 4, and 2% had 5. Let $X$ represent the number of occupants in a randomly chosen car.
   a. Find the probability mass function of $X$.
   b. Find $P(X \leq 2)$.
   c. Find $P(X > 3)$.
   d. Find $\mu_X$.
   e. Find $\sigma_X$.
   f. If 1000 cars are observed, how many cars we expect having at most 2 occupants?

Q2. It is known that screws produced by a certain company will be defective with probability .01, independently of each other. The company sells the screws in packages of 10 and offers a money-back guarantee that at most 1 of the 10 screws is defective. What proportion of packages sold must the company replace?

Q3. A large company has an inspection system for the batches of small compressors purchased from vendors. A batch typically contains 15 compressors. In the inspection system a random sample of 5 is selected and all are tested. Suppose there are 2 faulty compressors in the batch of 15.
   (a) What is the probability that for a given sample there will be 1 faulty compressor?
   (b) What is the probability that inspection will discover both faulty compressors?

Q4. There are two vacancies in a certain statistics department in the United States. Five individuals apply. Two have expertise in linear models and one has expertise in applied probability. The search committee is instructed to choose the two members randomly.
   (a) What is the probability that the two chosen are those with expertise in linear models?
   (b) What is the probability that from the two chosen, one has expertise in linear models and one has expertise in applied probability?

Q5. Three people toss a fair coin and the odd man pays for coffee. If the coins all turn up the same, they are tossed again. Find the probability that fewer than 4 tosses are needed.
Q6.
The number of flaws in bolts of cloth in textile manufacturing is assumed to be Poisson distributed with a mean of 0.1 flaw per square meter.
(a) What is the probability that there are two flaws in 1 square meter of cloth?
(b) What is the probability that there is one flaw in 10 square meters of cloth?
(c) What is the probability that there are no flaws in 20 square meters of cloth?
(d) What is the probability that there are at least two flaws in 10 square meters of cloth?